Schwartz 6.2

Taking x_1 as source, the retarded ~~int~~ propagator is

$$D(x_1, x_2) = \langle 0 | \phi_0(x_2) \phi_0(x_1) | 0 \rangle$$

$$= \langle 0 | \int \frac{d^3\vec{k}_1}{(2\pi)^3} \frac{d^3\vec{k}_2}{(2\pi)^3} \left[a_{k_2} e^{-ik_2 x_2} + a^\dagger_{k_2} e^{ik_2 x_2} \right]$$

$$\times \left[a_{k_1} e^{-ik_1 x_1} + a^\dagger_{k_1} e^{ik_1 x_1} \right] |0\rangle \frac{1}{\sqrt{2\omega_{k_1}}} \frac{1}{\sqrt{2\omega_{k_2}}}$$

$$= \int \frac{d^3\vec{k}_1}{(2\pi)^3} \frac{d^3\vec{k}_2}{(2\pi)^3} \langle 0 | a_{k_2} a^\dagger_{k_1} | 0 \rangle e^{-ik_2 x_2} e^{ik_1 x_1} \frac{1}{\sqrt{2\omega_{k_1}} \sqrt{2\omega_{k_2}}}$$

$$= \int \frac{d^3\vec{k}_1}{(2\pi)^3} \frac{d^3\vec{k}_2}{\cancel{(2\pi)^3}} \cancel{(2\pi)^3} \delta^3(\vec{k}_1 - \vec{k}_2) \frac{1}{\sqrt{2\omega_{k_1}} \sqrt{2\omega_{k_2}}} e^{-ik_2 x_2} e^{ik_1 x_1}$$

$$= \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} e^{-ik(x_2 - x_1)}$$

$$= \int \frac{d^4k}{(2\pi)^3} \frac{1}{2k_0} e^{-ik(x_2 - x_1)} \delta(k_0 - |\vec{k}|)$$

(propagating field have $m=0$)

Similarily, the advanced propagator would be

$$\int \frac{d^4k}{(2\pi)^3} \frac{1}{2k_0} e^{-ik(x_1 - x_2)} \delta(k_0 - |\vec{k}|)$$

The difference is that ~~for retard~~ we have $(x_2)_0 \geq (x_1)_0$ for retarded, and $(x_1)_0 \geq (x_2)_0$ for advanced propagator.

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